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Solutions of Higher Dimensional Gauss-Bonnet FRW Cosmology

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Abstract We examine the effect on cosmological evolution of adding a Gauss-Bonnet term to the standard Einstein-Hilbert action for a $(1+3)+d$ dimensional Friedman-Robertson-Walker (FRW) metric. By assuming that the additional dimensions compactify as a power law as the usual 3 spatial dimensions expand, we solve the resulting dynamical equations and find that the solution may be of either de Sitter or Kasner form depending upon whether the Gauss-Bonnet term or the Einstein term dominates.

Keywords Gauss Bonnet · Compactified extra dimensions · Friedman-Robertson-Walker

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1 Introduction

Ever since the introduction of extra dimensions by Kaluza and Klein, the notion that hidden extra dimensions may play a role in the dynamics of our usual four dimensional spacetime has received considerable attention (for a review see Applequist et. al. [1], [2]. However if these extra dimensions exist,

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experiments do provide constraints on the maximum current size (of extra dimensions) to be $\sim 100 \mu m$. [3] An idea that has been extensively studied is that these extra dimensions were once large but underwent dynamical compactification as the usual three spatial dimensions grew. This has been studied by Paul and Mukherjee [4] and Mohammadi [5] among others.

It is also widely thought that Einstein gravity is only a low energy effective field theory which requires modification at higher energy. A detailed review of this idea was given by Deser in [6]. One possible modification of the Einstein-Hilbert action comes from adding additional Lovelock terms. [7] This modification is attractive because it yields second-order, divergence free field equations as one would demand from a generally covariant theory of gravitation.[8] One may formulate Lovelock gravity theory as an expansion in powers of the curvature to obtain a zeroth-order constant term or cosmological constant, a first-order term that gives the usual scalar curvature which yields Einstein gravity, and a second-order term that is known as the Gauss-Bonnet term plus higher order terms.

This paper incorporates both of these ideas: compactification of the higher dimensions and the addition of a Gauss-Bonnet term to the action. We consider a dynamical compactification of a D -dimensional manifold to a maximally symmetric manifold of dimension d and an expanding FRW space-time of dimension 4 where we have modified the Einstein-Hilbert action by including a Gauss-Bonnet (GB) term.[9] This Gauss-Bonnet term can be interpreted as being a first order correction from string theory or simply a modification of Einstein gravity.

We will choose a power relation between the conformal factors of the three spatial dimensions and the d extra dimensions. This choice is motivated by the fact when the model is void of Gauss-Bonnet terms, the resulting field equations become exactly that of 4D FRW cosmology for arbitrary values of n and d , after one redefines the coupling and cosmological constant. [5]

This paper is organized as follows. In section 2 we present a general Einstein-Hilbert plus Gauss-Bonnet action in D -dimensions where the field equations are calculated and the correction to the FRW equation is given. In section 3, we define an “effective” pressure which an observer constrained to only the four dimensional volume would observe. This arises from the conservation equation, where this defined effective pressure yields an identical expression for the conservation equation to that of a 4D theory. In section 4, we find specific solutions for the $4 + d$ dimensional Einstein plus Gauss-Bonnet equations for cases when the Einstein terms or the Gauss-Bonnet terms dominate. Finally in section 5 we summarize our results.

2 Framework for Field Equations

In this paper we will follow the notation of Paul and Mukherjee[4] and Mohammadi [5] to express the Einstein-Hilbert action with an extra Gauss-Bonnet term as

$$S = \int d^D x \sqrt{-g} [R - \lambda - \epsilon \mathcal{G}] \quad (1)$$

where \mathcal{G} is a Gauss-Bonnet term ($\mathcal{G} \equiv R_{ABCD}R^{ABCD} - 4R_{AB}R^{AB} + R^2$). A variation of the action (1) with respect to g_{AB} produces the equation;

$$G_A{}^C + \lambda g_A{}^C + \epsilon \mathcal{G}_A{}^C = \frac{1}{\kappa} T_A{}^C \quad (2)$$

where κ is the $4 + d$ dimensional coupling constant and the Einstein and Gauss-Bonnet tensors, respectively, are

$$G_A{}^C = R_A{}^C - \frac{1}{2} g_A{}^C R \quad (3)$$

$$\begin{aligned} \mathcal{G}_A{}^C = & \frac{1}{2} (R_{BDEF} R^{BDEF} - 4R_{BD} R^{BD} + R^2) \delta_A{}^C \\ & - (2R_{BDEA} R^{BDEC} + 2R R_A{}^C - 4R_D{}^B R_{BA}{}^{DC} - 4R_B{}^C R_A{}^B). \end{aligned} \quad (4)$$

Furthermore we will assume that the stress-energy tensor will be that of a perfect fluid, thus it is of the form

$$T_\mu{}^\nu = \text{diag}[\rho(t), p(t), p(t), p(t), p_d(t), \dots, p_d(t)] \quad (5)$$

where $p_d(t)$ is the pressure on the higher dimensional compact manifold.

We will choose a metric ansatz:

$$\begin{aligned} ds^2 = & -dt^2 + a^2(t) \left[\frac{dr^2}{1 - Kr^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right] \\ & + b^2(t) \gamma_{mn}(y) dy^m dy^n \end{aligned} \quad (6)$$

where the extra dimensions are defined to be maximally symmetric such that the Riemann tensor for γ has the form $R_{abcd} = k(\gamma_{ac}\gamma_{bd} - \gamma_{ad}\gamma_{bc})$. In agreement with current observations we will consider the usual 3 spatial dimensions to be flat ($K = 0$) and also demand that the extra dimensions be flat ($k = 0$) as well in agreement with Mohammedi [5] that one finds unphysical properties for ρ and p if $k \neq 0$.

The metric leads to Riemann tensors of the following form (where both the dimensions are flat)

$$\begin{aligned} R_{a0a0} &= a\ddot{a}, \quad R_{abab} = a^2\dot{a}^2, \quad R_{m0m0} = b\ddot{b}, \\ R_{mama} &= a\dot{a}\dot{b}, \quad R_{abab} = b^2\dot{b}^2 \end{aligned} \quad (7)$$

where a, b are indices which run from 1...3 and m, n are indices which are in the extra dimensions. The Ricci Tensor and Ricci Scalar are

$$\begin{aligned} R_{00} &= 3\frac{\ddot{a}}{a} + d\frac{\ddot{b}}{b}, \quad R_{aa} = 2\dot{a}^2 + a\ddot{a} + d\frac{a\dot{a}\dot{b}}{b}, \\ R_{mm} &= 3\frac{b\dot{a}\dot{b}}{a} + (d-1)\dot{b}^2 + \ddot{b}b \end{aligned} \quad (8)$$

$$R = 6\frac{\ddot{a}}{a} + 2d\frac{\ddot{b}}{b} + 6d\frac{\dot{a}\dot{b}}{ab} + 6\frac{\dot{a}^2}{a^2} + d(d-1)\frac{\dot{b}^2}{b^2} \quad (9)$$

where d is the number of extra dimensions.

The full Einstein tensor may be expressed as having an Einstein term $G_\mu{}^\nu$ and a Gauss-Bonnet term $\mathcal{G}_\mu{}^\nu$. We make the assumption that the extra dimensions compactify as the 3 spatial dimensions expand as was done by Mohammadi [5]

$$b(t) \sim \frac{1}{a^n(t)} \quad (10)$$

where $n > 0$ in order to insure that the scale factor of the compact manifold is both dynamical and compactifies as a function of time. With this ansatz, the non-zero elements of the Einstein tensor take the form

$$G_0{}^0 = -\eta_1 \frac{\dot{a}^2}{a^2} \quad (11)$$

$$G_a{}^a = \eta_2 \frac{\ddot{a}}{a} + (\eta_1 - \eta_2^2) \frac{\dot{a}^2}{a^2} \quad (12)$$

$$G_m{}^m = \frac{1}{d} \frac{1}{n} \left[(2\eta_1 + 3\eta_2) \frac{\ddot{a}}{a} + [2(2\eta_1 + 3\eta_2) - d n (\eta_1 + 3\eta_2)] \frac{\dot{a}^2}{a^2} \right]. \quad (13)$$

where we defined the coefficients

$$\begin{aligned} \eta_1 &= \frac{1}{2} [6 + d n (d n - n - 6)] \\ \eta_2 &= (d n - 2) \end{aligned} \quad (14)$$

Note that when $D = 4$ (or $d = 0$) the equations become the well known FRW equations in four dimensions. In $4 + d$ dimensions, the Gauss-Bonnet terms become

$$\mathcal{G}_0{}^0 = \xi_1 \frac{\dot{a}^4}{a^4} \quad (15)$$

$$\mathcal{G}_a{}^a = \frac{1}{3} (\xi_1 - d n \xi_3) \frac{\dot{a}^4}{a^4} + \xi_3 \frac{\ddot{a} \dot{a}^2}{a^3} \quad (16)$$

$$\mathcal{G}_m{}^m = -(\xi_1 + \xi_3) \frac{\dot{a}^4}{a^4} + \frac{1}{d} \frac{1}{n} (3 \xi_3 + 4 \xi_1) \frac{\ddot{a} \dot{a}^2}{a^3} \quad (17)$$

where we defined the coefficients

$$\begin{aligned} \xi_1 &= -d n \left[(d-1) n \left[\frac{1}{2} (d-2) n [(d-3) n - 12] + 18 \right] - 12 \right] \\ \xi_3 &= -2 d n \left[(d-1) n [(d-2) n - 6] + 6 \right] \end{aligned} \quad (18)$$

where the constants η_i and ξ_i depend upon the values of n and d as defined above. Note that if we force the number of extra dimensions to zero ($d \rightarrow 0$) then the Gauss-Bonnet terms vanish for $\mathcal{G}_0{}^0$ and $\mathcal{G}_a{}^a$ as one would expect in four dimensions.

3 Effective Pressure and the Field Equations

For the case of both spaces having flat curvature, the D -dimensional Friedmann-Robertson-Walker (FRW) equations (2) take the form

$$\frac{\rho}{2\kappa} = \eta_1 \frac{\dot{a}^2}{a^2} + \epsilon \xi_1 \frac{\dot{a}^4}{a^4} \quad (19)$$

$$\frac{p}{2\kappa} = \left[\eta_2 \frac{\ddot{a}}{a} + (\eta_1 - \eta_2) \frac{\dot{a}^2}{a^2} \right] + \epsilon \left[\frac{1}{3} (\xi_1 - d n \xi_3) \frac{\dot{a}^4}{a^4} + \xi_3 \frac{\ddot{a} \dot{a}^2}{a^3} \right] \quad (20)$$

$$\begin{aligned} \frac{p_d}{2\kappa} = \frac{1}{d n} & \left[(2\eta_1 + 3\eta_2) \frac{\ddot{a}}{a} + [2(2\eta_1 + 3\eta_2) - d n (\eta_1 + 3\eta_2)] \frac{\dot{a}^2}{a^2} \right] \\ & - \epsilon \left[(\xi_1 + \xi_3) \frac{\dot{a}^4}{a^4} - \frac{1}{d n} (4 \xi_1 + 3 \xi_3) \frac{\ddot{a} \dot{a}^2}{a^3} \right] \end{aligned} \quad (21)$$

where we have set $\lambda = 0$. Together with these Einstein equations, we also demand that the conservation equation hold for the stress-energy tensor ($\nabla_\mu T^\mu{}_\nu = 0$) or

$$\left\{ \frac{d}{dt}(a^3 \rho) + p \frac{d}{dt}(a^3) \right\} + d a^3 \frac{\dot{b}}{b} (\rho + p_d) = 0 \quad (22)$$

Using the assumption that $b = 1/a^n$, this becomes

$$\frac{d}{dt}(a^3 \rho) + \tilde{p} \frac{d}{dt}(a^3) = 0 \quad (23)$$

which by simple algebra may be written in the more familiar form

$$\dot{\rho} + 3 \frac{\dot{a}}{a} (\rho + \tilde{p}) = 0. \quad (24)$$

Note that this is simply a statement that $dE = -P dV$ where we have defined an “effective” pressure (\tilde{p}) [5] which an observer constrained to exist only upon the “usual” 3 spatial dimensions would see as

$$\tilde{p} = p - \frac{1}{3} d n (\rho + p_d). \quad (25)$$

As was pointed out by Mohammadi [5], this effective pressure can be negative for positive values of ρ , p , and p_d . The effective pressure can be easily computed from the d -dimensional FRW equations (19)-(21) and is given by the relation

$$\frac{\tilde{p}}{2\kappa} = -\frac{1}{3} \eta_1 \left(2 \frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} \right) - \frac{1}{3} \epsilon \xi_1 \left(4 \frac{\ddot{a} \dot{a}^2}{a^3} - \frac{\dot{a}^4}{a^4} \right) \quad (26)$$

Note that the above field equations (19)-(21), (26) become the same as [5] in the limit where $\epsilon \rightarrow 0$ (i.e. no Gauss-Bonnet terms). By redefining the coupling and cosmological constant in eqns. (19) and (26), one recovers standard 4-D FRW cosmology for arbitrary values of n and d in the $\epsilon \rightarrow 0$ limit.

4 The General Solutions of the GB FRW Field Equations

The effective D -dimensional FRW equations and the conservation equation now read

$$0 = \dot{\rho} + 3\frac{\dot{a}}{a}(\rho + \tilde{p}) \quad (27)$$

$$\frac{\rho}{2\kappa} = \eta_1 \frac{\dot{a}^2}{a^2} + \epsilon \xi_1 \frac{\dot{a}^4}{a^4} \quad (28)$$

$$\frac{\tilde{p}}{2\kappa} = -\frac{1}{3}\eta_1 \left(2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} \right) - \frac{1}{3}\epsilon \xi_1 \left(4\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2} \right) \frac{\dot{a}^2}{a^2} \quad (29)$$

$$\begin{aligned} \frac{p_d}{2\kappa} = & \frac{1}{d} \frac{1}{n} \left[(2\eta_1 + 3\eta_2) \left(\frac{\ddot{a}}{a} + 2\frac{\dot{a}^2}{a^2} \right) - d n (\eta_1 + 3\eta_2) \frac{\dot{a}^2}{a^2} \right] \\ & + \epsilon \frac{1}{d} \frac{1}{n} \left[3(\xi_1 + \xi_3) \left(\frac{\ddot{a}}{a} - \frac{1}{3} d n \frac{\dot{a}^2}{a^2} \right) + \xi_1 \frac{\ddot{a}}{a} \right] \frac{\dot{a}^2}{a^2} \end{aligned} \quad (30)$$

One can easily show that (27) is trivially satisfied when (28) and (29) are employed. This is exactly like that of 4D FRW cosmology where the variables are left underdetermined by the field equations and one usually chooses an equation of state (EoS) of the form

$$\tilde{p} = w \rho \quad (31)$$

to proceed. Notice that the parameter w can in general be time-dependent. If w is in fact constant, (27) can be integrated yielding

$$\rho(t) = \rho_0 a^{-3(1+w)} \quad (32)$$

Using (31) and eliminating ρ and \tilde{p} from the field equations, we obtain an expression of the form

$$(1+w) \left(\eta_1 + \epsilon \xi_1 \frac{\dot{a}^2}{a^2} \right) \frac{\dot{a}^2}{a^2} = -\frac{2}{3} \left(\eta_1 + 2\epsilon \xi_1 \frac{\dot{a}^2}{a^2} \right) \frac{d}{dt} \left(\frac{\dot{a}}{a} \right) \quad (33)$$

The behavior of the scale factor is determined by this expression. As is obvious from (33), the scale factor is dependent on the value of the parameter w as one would expect. In the following subsections, we will examine the solutions of this equation for cases when parameter is uniquely $w = -1$ and the general case, when $w \neq -1$.

4.1 Solutions for $w = -1$

If $w = -1$, the equation of state (33) reduces to

$$0 = \left(\eta_1 + 2\epsilon \xi_1 \frac{\dot{a}^2}{a^2} \right) \frac{d}{dt} \left(\frac{\dot{a}}{a} \right) \quad (34)$$

This equation has two solutions depending on which bracket is equated to zero. In either case, we find a de Sitter-type solution given by the expressions

$$a(t) = a_0 e^{Ht} \quad (35)$$

where H is the Hubble constant which is obtained from the remaining field equations. Plugging $a(t)$ into (28) yields a value for the Hubble parameter of the form

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = -\frac{\eta_1}{2\epsilon\xi_1} \left[1 \pm \sqrt{1 + \frac{4\epsilon\xi_1}{\eta_1^2} \cdot \frac{\rho}{2\kappa}} \right]. \quad (36)$$

Notice that when the vacuum-energy density is chosen such that the radical vanishes, we find a result corresponding to what one obtains when the first bracket is set equal to zero. Hence, the first solution is simply a special case of the second solution.

In the small epsilon limit, when the Gauss-Bonnet contribution offers a small correction to the Einstein-Hilbert term, we find

$$H^2 \simeq \frac{\rho}{2\kappa\eta_1} \left[1 - \epsilon \frac{\xi_1}{\eta_1^2} \frac{\rho}{2\kappa} \right] \quad (37)$$

to $\mathcal{O}(\epsilon)$. Note that we kept only the negative root of (36) so that the value of the Hubble constant approaches that of 4D FRW cosmology in this small ϵ limit.

In the large epsilon limit, when the Gauss-Bonnet term dominates over the Einstein term, we find a value for the Hubble constant of the form

$$H^2 \simeq \pm \frac{1}{\beta} \sqrt{\frac{\rho}{2\kappa\eta_1}} - \frac{1}{2\beta^2} \quad (38)$$

where we defined the parameter

$$\beta = \sqrt{\frac{\epsilon\xi_1}{\eta_1}} \quad (39)$$

and kept terms to $\mathcal{O}(1/\epsilon)$.

We can eliminate the expansion factor and obtain a higher dimensional equation relating both p_d and ρ . This EoS-like expression takes the form

$$(p_d - \psi_0\rho) + \frac{\epsilon}{2} \frac{\sigma}{(\psi_0 - \chi_0)} (p_d - \chi_0\rho)^2 = 0 \quad (40)$$

where we defined the constants

$$\begin{aligned} \psi_0 &= -\frac{1}{\eta_1} \left[\left(1 - \frac{3}{d n}\right) (2\eta_1 + 3\eta_2) - \eta_1 \right] \\ \chi_0 &= -\frac{1}{\xi_1} \left[\left(1 - \frac{3}{d n}\right) (\xi_1 + \xi_3) - \frac{1}{d n} \xi_1 \right]. \end{aligned} \quad (41)$$

Notice that (40) is inherently *non-linear* as one might expect from an exact solution to Gauss-Bonnet FRW cosmology. Also notice that when $d n = 3$, which corresponds to a constant volume $d + 4$ dimensional Universe, (41) dramatically simplifies yielding the fixed parameters $\psi_0 = 1$ and $\chi_0 = 1/3$.

4.2 Solution for $w \neq -1$

In this subsection, we investigate the behavior of the scale factor for the general case when w is left as a free parameter. By integrating the equation of state, eqn. (33)

$$\beta \frac{\dot{a}}{a} = \tan \left[\frac{1}{\beta} \left(\frac{1}{\dot{a}/a} - \frac{3}{2}(1+w)t \right) \right] \quad (42)$$

where we again defined $\beta = \sqrt{\epsilon \xi_1 / \eta_1}$. In this section, we can explore this behavior by investigating (42) when the Gauss Bonnet term is small and when it dominates over the Einsteinian term.

4.2.1 Small epsilon regime

In the small ϵ regime, one can expand the equation of state (42) by taking the arctan of each side and taking the expansion for small x . Keeping only the lowest order contribution in the expansion, it becomes

$$\beta^2 \frac{\dot{a}^2}{a^2} + \frac{3}{2}(1+w)t \frac{\dot{a}}{a} - 1 \simeq 0. \quad (43)$$

Solving for the Hubble parameter, we obtain

$$\frac{\dot{a}}{a} \simeq -\frac{3}{4\beta^2}(1+w)t \left[1 \pm \sqrt{1 + \left[\frac{4\beta}{3(1+w)t} \right]^2} \right]. \quad (44)$$

Again, expanding the square root in the small ϵ ($\sim \beta^2$) limit and keeping the first order ϵ contribution, one obtains

$$H(t) = \frac{\dot{a}}{a} \simeq \frac{2}{3(1+w)t} \left[1 - \epsilon \frac{\xi_1}{\eta_1} \left[\frac{2}{3(1+w)t} \right]^2 \right] \quad (45)$$

where we again kept only the negative root in order to arrive at 4D FRW cosmology in this small ϵ limit. Using this result, we integrate (45) and obtain a value for the scale factor of the form

$$a(t) \simeq \mu t^{2/3(1+w)} \left[1 + \epsilon \frac{\xi_1}{2\eta_1} \left[\frac{2}{3(1+w)} \right]^3 \frac{1}{t^2} \right] \quad (46)$$

to first order in ϵ . Note that in the large time limit ($t \rightarrow \infty$) the scale factor tends to its zeroth-order value. Hence, we see that the Gauss-Bonnet contribution becomes vanishingly insignificant for late cosmological times as one would expect.

As a consistency check, we insert (32), (45), and (46) into (28) and find that the time dependence does in fact vanish to first order in ϵ . We obtain a relation on the coefficients given by

$$\mu = \left[\left[\frac{3}{2}(1+w) \right]^2 \frac{\rho_0}{2\kappa\eta_1} \right]^{1/3(1+w)} \quad (47)$$

which agrees with the value one obtains from 4D FRW cosmology.

4.2.2 Large ϵ regime

Interestingly, because the first order term in the the expansions of $\tan(x)$ and $\arctan(x)$ are both linear in x ; the expansion for when the Gauss-Bonnet term dominates (large ϵ or equivalently when $t \ll \beta$), (42) is the same as for small ϵ (43)

$$\beta \frac{\dot{a}}{a} = \frac{1}{\beta} \left(\frac{1}{\dot{a}/a} - \frac{3}{2}(1+w)t \right). \quad (48)$$

The difference between the large and small ϵ cases is that in the large ϵ case one does not expand the radical as was done in (45), instead one may simply express the equation in terms of H as

$$H^2 + \left[\frac{3t}{2\beta^2}(1+w) \right] H - \frac{1}{\beta^2} = 0. \quad (49)$$

If solves this quadratic and keeps the terms large in $1/\epsilon$ (the terms that dominate when GB is large), one obtains

$$a(t) \approx a_0 \text{Exp} \left[\frac{t}{\beta} - \frac{3t^2}{8\beta^2}(1+w) + \mathcal{O} \left(\frac{t^3}{\beta^3} \right) \right]. \quad (50)$$

Note that when the Gauss Bonnet terms dominate the form of $a(t)$ is de Sitter like.

5 Conclusion

In conclusion, we have studied the Friedman equations for an Einstein plus Gauss-Bonnet action in $4 + d$ dimensions. We furthermore demanded that the extra dimensions compactify as $a(t) \sim b(t)^{-n}$ where $n > 0$. In Section 4, we solved the field equations for cases when $w = -1$ and $w \neq -1$. We found solutions for both of these cases when the Gauss-Bonnet term dominates the Einstein term early time cosmology and when the Gauss-Bonnet term is subdominant to the Einstein term (late time cosmology). We have found that when Einstein gravity dominates, the conformal factor is a Kasner solution with a small Gauss-Bonnet correction. However, when the Gauss-Bonnet term dominates, a de Sitter type solution is obtained indicating that the Gauss-Bonnet term gives rise to nontrivial corrections of the scale factor $a(t)$. One may conclude from this analysis that if the Gauss-Bonnet term is dominant, the compactifying extra dimensions can be thought of as playing the role of a “cosmological constant” forcing $a(t)$ to behave as a de Sitter solution even though no such constant is present in the action.

The natural question which arises from this paper is when does this model of compactifying internal dimensions with extra Gauss-Bonnet terms physically representative of our universe. Without further modification of the paper, it seems to be applicable only to early universe inflation as our model tends towards a Kasner solution in late time. In order to agree with current cosmological observations, the model should allow the extra dimensions to

compactify much slower than the three spatial dimensions grow to be in order to agree with the current limits on the running of the coupling constants.

This paper leads to several intriguing questions. First, does this same effect of compactifying extra dimensions leading to an effective cosmological constant exist in alternate models such as brane models like that of Randall-Sundrum [11] or Narain et al [12]. If so, then could this be used to generate an inflationary cosmology without the use of a scalar field [13]. A second and central issue to explore would be to relax the explicit relation between $a(t)$ and $b(t)$, could one then solve the resulting field equations? A possibility we are examining seems to require a numerical study.

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